A Nonlinear OTA Simulation Model for the Design of a Switched-Capacitor DSM

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***Abstract*—This paper proposes a nonlinear simulation model of an operational transconductance amplifier (OTA) for switched-capacitor (SC) delta-sigma modulators (DSMs). A worst-case condition is assumed to characterize the nonlinear behavior of the OTA gain. Under this assumption, the OTA input-output relationship is analytically derived and incorporated into a Simulink-based SC DSM model. The resulting simulation outcomes are then compared with experimental measurements obtained from a fabricated SC DSM chip, thereby validating the effectiveness of the proposed modeling approach and methodology.**

***Keywords*—Delta-Sigma modulator (DSM), switched-capacitor circuit, hybrid sturdy MASH DSM, operational transconductance amplifier**

I. Conjecture

The OTA polynomial gain model is normalized as

$$y(x) = \frac{A_v(v_o)}{A_0} = \left(1 + \sum_{k=1}^{K} q_{2k} x^{2k}\right) \tag{1}$$

where $x = v_o/A$. We conjecture that, for the $2K$th-order normalized OTA polynomial gain model (1), the maximum curvature at $x = 0$ will be $2K$. The corresponding OTA polynomial gain model is

$$A_v(v_o) = \frac{dv_o}{dv_i} = A_0\left(1 - \left(\frac{v_o}{A}\right)^2\right)^K. \tag{2}$$

The iterative formula from [1] is applied to derive the input-output relationship of order $2K$.

$$v_i = \frac{A}{A_0}\int \frac{dx}{(1-x^2)^K} = \frac{A}{A_0}\left\{\frac{x}{(2K-2)(1-x^2)^{K-1}} + \frac{2K-3}{2K-2}\int \frac{dx}{(1-x^2)^{K-1}}\right\}. \tag{3}$$

II. Hybrid Sturdy MASH-21 DSM

Fig. 2 shows the simulated output spectrum of Y when the OTA nonlinear model uses $K = 8$. The corresponding normalized nonlinear model for $K = 8$, derived from (3), is given by

$$\int \frac{dx}{(1-x^2)^8} = \frac{x}{14(1-x^2)^7} + \frac{13}{14}\left[\frac{x}{12(1-x^2)^6} + \frac{11}{12}\left\{\frac{x}{10(1-x^2)^5} + \frac{9}{10}\left[\frac{x}{8(1-x^2)^4} + \frac{7}{8}\left(\frac{x}{6(1-x^2)^3} + \frac{5x}{24(1-x^2)^2} + \frac{5x}{16(1-x^2)}\right)\right]\right\}\right] + \frac{3003\tanh^{-1}(x)}{14336}$$

III. Comparison with Experimental Results

By comparing the simulated spectrum in Fig. 2 with the measured spectrum in Fig. 3, it is observed that the total harmonic distortions (THDs) are closely matched, and the signal-to-noise-and-distortion ratios (SNDRs) differ by only 0.63 dB. The signal-to-noise ratio (SNR), however, exhibits a larger difference of 1.17 dB between the two spectra. Since input-referred noise is not included in the simulations, this SNR discrepancy is considered acceptable. It is also noted that the proposed nonlinear model employs only the OTA output saturation level A and the maximum gain A_0 as parameters in (3). Despite this simplicity, the close agreement between the simulated and measured spectra demonstrates the effectiveness of the proposed modeling approach.

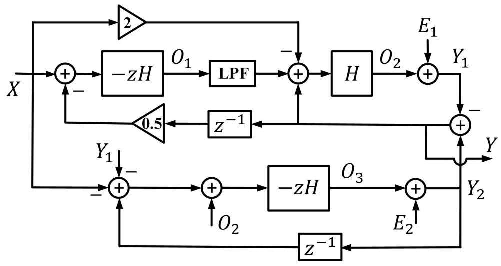

Fig. 1. The block diagram of a HSMASH-21 DSM.

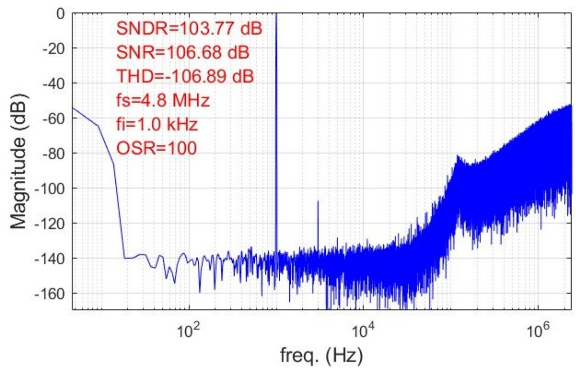

Fig. 2. The simulated spectrum with the first OTA gain equal to 62.4 dB, and the second and the third OTAs gain equal to 38.8 dB. All OTAs use nonlinear gain model of $K = 8$

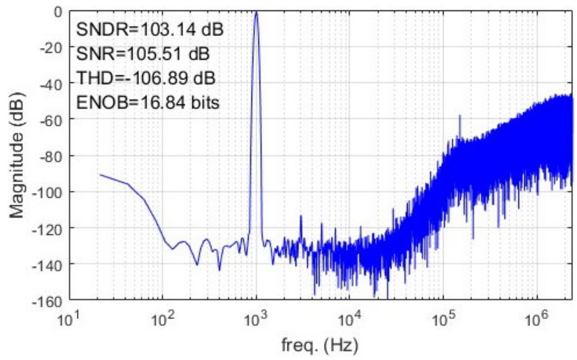

Fig. 3. Measured spectrum of the HSMASH-21 DSM chip.

References

[1] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Product, 7th ed: Formula 2.149*. Burlington, MA: Academic Press, 2007.