PCB Modeled by a Lossy Three-Conductor Transmission Line

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SUMMARY

A lossy three-conductor transmission line as a representation of a printed circuit boards in electromagnetic compatibility context is modeled. It is terminated by linear loads. The model extends the consideration of C. Paul for lossless transmission lines. A hyperbolic system for the currents and voltages of the line is formulated. The corresponding mixed problem for this system is solved by analytical transformations and reduced to a initial value problem on the boundary of the domain.

Typical transmission-line structures are cables and printed circuit boards (PCB). A PCB in transmission line context, consists of a dielectric board with conductors (or lands) on it that interconnect electronic devices (both digital and analog). Multiconductor transmission lines are sets of *n*+1 parallel conductors that transmit signals between different points in a circuit, such as a source and a load [1]. Transverse electromagnetic mode (TEM) of propagation is assumed dominant on transmission lines.

Our previous studies include analyses of three-conductor lossless transmission lines terminated by linear resistive loads and terminated by nonlinear resistive loads, respectively. In this paper, we consider a model of electromagnetic compatibility of PCB based on three-conductor transmission line based on the Clayton Paul results [1], [2]. Our task is obtaining the complete solution for the voltages and currents of the three-conductor transmission line. Following the technique from [3] we obtain a general solution of the system modelling pairwise interacting three-conductor transmission line. We proceed from the three-conductor line shown in the Fig. 1. The ground symbol is a denotation of the third conductor for the line voltages reference conductor is referred to as the Receptor circuits.

The voltages with respect to the reference conductor $u_k(x,t)$ $(k=1,2)$ and currents of each circuit $i_k(x,t)$ $(k=1,2)$ are functions of position *x* and time *t*. Considering lossy TEM mode of propagation we have $C\frac{\partial u(x,t)}{\partial t}+\frac{\partial i(x,t)}{\partial x}+Gu(x,t)=0,\ and\ \ L\frac{\partial i(x,t)}{\partial t}+\frac{\partial u(x,t)}{\partial x}+Ri(x,t)=0$. We solve the following transmission line system

$$\left|\begin{array}{l}
(C_{GEN}+C_m)\dfrac{\partial u_{GEN}(x,t)}{\partial t}+\dfrac{\partial i_{GEN}(x,t)}{\partial x}+G_{GEN}u_{GEN}(x,t)=C_m\dfrac{\partial u_{REC}(x,t)}{\partial t}\\
L_{GEN}\dfrac{\partial i_{GEN}(x,t)}{\partial t}+\dfrac{\partial u_{GEN}(x,t)}{\partial x}+R_{GEN}i_{GEN}(x,t)=-L_m\dfrac{\partial i_{REC}(x,t)}{\partial t}\\
(C_{REC}+C_m)\dfrac{\partial u_{REC}(x,t)}{\partial t}+\dfrac{\partial i_{REC}(x,t)}{\partial x}+G_{REC}u_{REC}(x,t)=C_m\dfrac{\partial u_{GEN}(x,t)}{\partial t}\\
L_{REC}\dfrac{\partial i_{REC}(x,t)}{\partial t}+\dfrac{\partial u_{REC}(x,t)}{\partial x}+R_{REC}i_{REC}(x,t)=-L_m\dfrac{\partial i_{GEN}(x,t)}{\partial t}
\end{array}\right.$$

with respective boundary and initial conditions. This system extends to the model introduced by C. Paul [1], [2] accounting for the losses along the line. Thie leads to more complicated system of equations of motion. We transform the mixed problem (i.e. the system with boundary and initial conditions) to a delay functional system with initial conditions only; the obtained system of functional equations is with two different delays. With this result we reduced the mixed problem to a functional system with initial conditions only. Such system could be solved by a fixed point method and the existence of a periodic solution could be given – subject to a next paper.

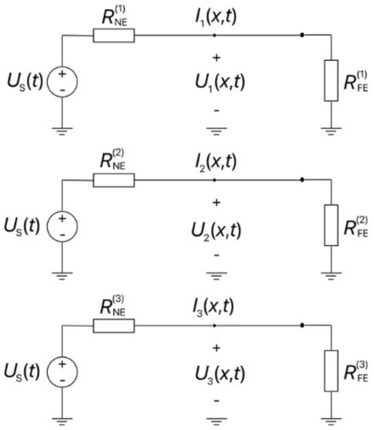

Fig. 1. Equivalent circuit of the three-conductor transmission line.

REFERENCES

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